NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2014

12-04-2014 (Online-3)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. A spherically symmetric charge distribution is characterised by a charge density having the following variation:

$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$ for r < R

$\rho(r) = 0$ for $r \geq R$

Where r is the distance from the centre of the charge distribution and ρ_0 is a constant. The electric field at an internal point (r < R) is :

(A) $\frac{\rho_0}{4\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$ (B) $\frac{\rho_0}{12 \in_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$ (C*) $\frac{\rho_0}{\in_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$ (D) $\frac{\rho_0}{3 \in_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

Sol. $\oint \varepsilon ds = \frac{\Sigma a}{\varepsilon_0}$

$\varepsilon(4\pi r^2) = \rho \cdot \int_0^r \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$

$4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^2}{4R}\right]_0^q = \varepsilon(4\pi r^2)$

$\varepsilon = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

2. A spring of unstretched length ℓ has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is :

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{3}mv^2$ (C) mv^2 (D*) $\frac{1}{6}mv^2$

Sol. $KE = \int \frac{1}{2} dm \left(\frac{v}{\ell} x\right)^2$

$= \frac{1}{2}\int \frac{m}{\ell} dx \frac{v^2}{\ell^2} x^2$

$= \frac{mv^2}{2\ell^2}\left(\frac{x^3}{3}\right)_0^\ell$

$= \frac{mv^2}{2\ell^3} \times \frac{\ell^3}{3}$

$= \frac{mv^2}{6}$

3. A sinusoidal voltage V(t) = 100 sin (500t) is applied across a pure inductance of L = 0.02 H. The current through the coil is:

(A) – 10 cos (500t) (B) 10 cos(500 t) (C) 10 sin (500t) (D*) – 10 sin (500t)

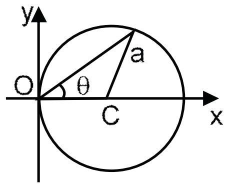
Sol. $x_L = \omega L$

$$= \frac{2\pi \times 500}{500 \times \frac{.02}{100}}$$

i = 10 sin [ωt – π/2]

= – 10 cos 500t $\Rightarrow$ 10

4. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The centre of circle is marked by 'C'. The angular momentum from the origin O can be written as:



(A) vacos2θ (B*) va(1 + cos 2θ) (C) va (D) va(1 + cosθ)

Sol. (v cos θ) × 2 turns

2cos va –1 = cos 2w

2rv $\cos^2$v

va (1 + cos 2θ)

5. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is:

(A*) 10°C (B) 20°C (C) 15°C (D) 25°C

So. $\frac{10}{10} = K\,[55 - T]$

$\frac{8}{10} = K\,[46 - T]$

T = 10°C

for average, interval should be small.

$\frac{60-50}{10} = K\,[55 - T]$

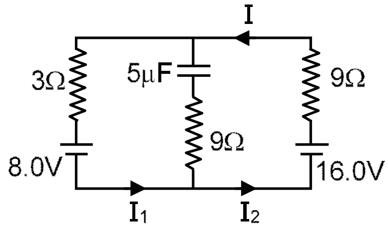
$\frac{18}{10} = K\,[51 - T]$

$\frac{10 \times 20}{10 \times 18} = \frac{55 - T}{51 - T}$

510 – 101 = 55 × 9 – 91

15 = 1

6. The circuit shown here has two batteries of 8.0 V and 16.0 V and three resistors 3Ω, 9Ω and 9 Ω and a capacitor 5.0 μF.

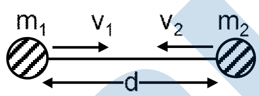


How much is the current I in the circuit in steady state?

(A) 1.6 A (B) 0.25 A (C) 2.5 A (D*) 0.67 A

Sol. $\frac{8}{12}\cdot\frac{2}{3} = 0.67$

7. Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd' ? (Speed of m_1 is v_1 and that of m_2 is v_2) :



(A) $v_1 = v_2$

(B*) $v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

(C) $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(D) $v_1 = m_2\sqrt{\frac{2G}{m_1}}$

$v_2 = m_1\sqrt{\frac{2G}{m_2}}$

Sol. From M.E conservation

$$O = -\frac{GM_1M_2}{d} + KE$$

$$KE = \frac{GM_1M_2}{d}$$

Since momentum is constant

So $KE \propto \frac{1}{m}$

$$\text{K.E. of } m_1 = \left(\frac{m_2}{m_1+m_2}\right)\frac{GM_1M_2}{d} = \frac{1}{2}m_1v_1^2$$

$$v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

$$\text{K.E. of } m_2 = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{GM_1M_2}{d}\right) = \frac{1}{2}m_2v_2^2$$

$$v = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

8. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is:

(A) Cl_2 (B) F_2 (C) O_2 (D*) H_2

Sol. $\frac{1}{3}\rho v_{rms}^2 = p$ $\quad pv = \frac{MRT}{M_0}$

$v_{rms} = \sqrt{\frac{3p}{\rho}}$ $\quad \frac{p}{\rho} = \frac{RT}{M_0}$

$$1930 = \sqrt{\frac{3RT}{M_0}}$$

$$\frac{3\times 8.314\times 300}{1930\times 1930}$$

9. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants?

(A) $a - bx + cx^2$ (B) bx^2 (C*) $-bx$ (D) $a + bx - cx^2$

10. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be:

(A) 70% and – 10°C (B) 20% and – 43° C

(C) 50% and – 20°C (D*) 40% and – 33%°C

Sol. Q_1 = 1000 J

Q_2 = 600

$$\eta = 1 - \frac{Q_2}{Q_1}$$

= 40%

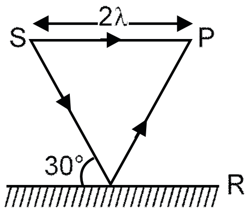
$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

$$\frac{400}{T_2} = \frac{1000}{600}$$

T_2 = 240 K – 273 = – 33°C

11. Interference pattern is observed at 'P' due to superimposition of two rays coming out from a source 'S' as shown in the figure. The value of 'ℓ' for which maxima is obtained at 'P' is:

(R is perfect reflecting surface):



(A) $\ell = \frac{(2n-1)\lambda}{\sqrt{3}-1}$ (B) $\ell = \frac{2n\lambda}{\sqrt{3}-1}$ (C*) $\ell = \frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$ (D) $\ell = \frac{(2n-1)\lambda}{2(\sqrt{3}-1)}$

Sol. x cos30° = ℓ

$x\frac{\sqrt{3}}{2} = \ell$ $\qquad x = \frac{\sqrt{2\ell}}{\sqrt{3}}$

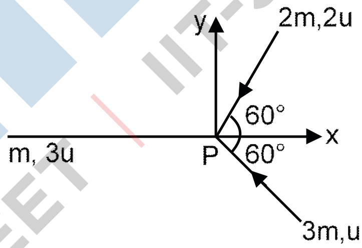
$\frac{2\ell}{\sqrt{3}} + \frac{2\ell}{\sqrt{3}} + \frac{\lambda}{2} - 2\ell = n\lambda$

$n\lambda - \frac{\lambda}{2}$

$2\ell\left(\frac{2}{\sqrt{3}} - 1\right) = (2n-1)\frac{\lambda}{2}$

$2\ell(2-\sqrt{3}) = \frac{(2n-1)\lambda\sqrt{3}}{2}$

12. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be:



(A*) $\frac{u}{12}(-\hat{i} - \sqrt{3}\hat{j})$ (B) $\frac{u}{12}(-\hat{i} + \sqrt{3}\hat{j})$ (C) $\frac{u}{12}(\hat{i} + \sqrt{3}\hat{j})$ (D) $\frac{u}{12}(\hat{i} - \sqrt{3}\hat{j})$

Sol. $p_i = p_y$

$m3u\hat{i} + 3m[-u\cos 60°\hat{i} + u\sin 60°\hat{j}]$

$+ 2m[-2u\cos 60°\hat{i} - 2u\sin 60°\hat{j}]$

$3mu\hat{i} + 3m\left[-\frac{v}{2}\hat{i} + \frac{0\sqrt{3}}{2}\hat{j}\right] + 2m\left[-\frac{2u}{2}\hat{i} - \frac{2u\sqrt{3}}{2}\hat{j}\right]$

$3u - \frac{3v}{2} - 2u$ $\frac{3\sqrt{3}}{2}\hat{j} - 2\sqrt{3}$

$-\frac{u}{2}$ $-\frac{\sqrt{3}}{2}\hat{j}$

13. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation:

$K(x) = K_0 + \lambda x$ (λ = a constant)

The capacitance C, of this capacitor, would be related to its 'vacuum' capacitance C_0 as per the relation:

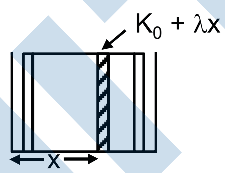
(A) $C = \frac{\lambda}{d \cdot \ell n(1 + K_0/\lambda d)} C_0$

(B) $C = \frac{\lambda}{d \cdot \ell n(1 + K_0 \lambda d)} C_0$

(C*) $C = \frac{\lambda d}{\ell n(1 + \lambda d/K_0)} C_0$

(D) $C = \frac{\lambda d}{\ell n(1 + K_0 \lambda d)} C_0$

Sol. $d_C = \frac{(K_0 + \lambda_x)A}{dx}$



$\int \frac{1}{d_C} = \int_0^{\phi} \frac{dx}{A \cdot (K_0 + \lambda_x)}$

$C_0 = \frac{\varepsilon_0 A}{d}$ $\frac{1}{\lambda} \int_{K_0}^{K_0 + \lambda d} \frac{dt}{At}$

$K_0 + \lambda x = t$ $\lambda dx = dt$

λA $\frac{1}{\lambda A} \ell n \left[\frac{K_0 + \lambda d}{K_0}\right]$

14. A 4 g bullet is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately?

(A) 0.569 m (B*) 0.758 m (C) 0.379 m (D) 0.19 m

Sol. $\frac{4}{1000} \times 300 = \frac{0.8}{10} v$

$v = \frac{3}{2} = 1.5 \text{ m/s}$

$v^2 - Cl^2 = 2as$

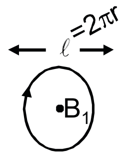
$\frac{g^3}{4} = 2 \times 3 \times 5$

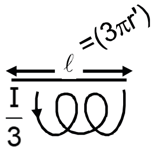
$\frac{3}{8} = 5 = 0.375 \text{ m}$

15. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current I passes through it. The second wire is bent into a coil with three identical loops adjacent to each other and produces magnetic field B_2 at the centre of the loops when current I/3 passes through it. The ratio B: B_2 is:

(A) 1: 1 (B*) 1: 3 (C) 9: 1 (D) 1: 9

Sol. $r = \frac{\ell}{2\pi}$





$r' = \frac{\ell}{6\pi}$

$B_1 = \frac{\mu_0 I}{2\pi\ell} \times 2\pi$ $\qquad$ $B_2 = 3 \cdot \frac{\mu_0 I/3}{2\pi\ell} \cdot 6\pi$

$= \frac{\mu_0 I}{\ell}$ $\qquad$ $= 3 \cdot \frac{\mu_0 I}{\ell} \cdot \frac{1}{3} \cdot 3$

$= 3$

$B_1 : B_2 = 1 : 3$

16. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is:

(A) $\frac{\mu_0 \in_0}{c^2} \frac{G}{he^2}$ $\qquad$ (B) $\frac{ch}{2\pi \in_0^2}$

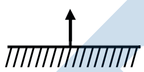
(C*) $\frac{e^2}{2\pi \in_0 Gm_e^2}$ (m_e = mass of electron) $\qquad$ (D) $\frac{2\pi\sqrt{\mu_0 \in_0}}{ce^2} \frac{h}{G}$

Sol. A dimensionless & unitless terms will have same value is all systems.

17. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground, of frequency nearly: (speed of sound = 343 m/s)

(A) 1800 Hz $\qquad$ (B*) 2500 Hz $\qquad$ (C) 2150 Hz $\qquad$ (D) 2400 Hz

Sol. v↓O source $\qquad$ $\left|\frac{343}{343+v}\right|1800$



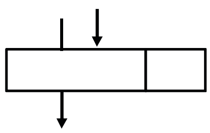
$343 = 1.2 \times 343 + 1.2v = 57.16$

$\left|\frac{343+57}{343}\right| \times 2150$

18. Steel ruptures when a shear of 3.5×10^8 N m^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly:

(A) 2.7×10^4 N $\qquad$ (B*) 3.3×10^4 N $\qquad$ (C) 1.4×10^4 N $\qquad$ (D) 1.1×10^4 N

Sol. $\frac{F}{A}$ = stress = 3.5×10^8 N/m^2



$A = 2\pi rt$

$= 2\pi \times \frac{1}{2} \times 0.3$

$= 0.3\pi \times 10^{-4}$ m^2

$F = A \times \text{stress}$

$= 0.3\pi \times 10^{-4} \times 3.5 \times 10^{8}$

$= 1.05\ \pi \times 10^{4}$

$= 3.3 \times 10^{4}$ N

19. A piece of bone of an animal from a ruin is found to have ^{14}C activity of 12 disintegrations per minute per gm of its carbon content. The ^{14}C activity of a living animal is 16 disintegrations per minute per gm. How long ago nearly did the animal die? (Given half life of ^{14}C is $t_{1/2}$ = 5760 years):

(A) 1672 years (B) 3291 years (C*) 2391 years (D) 4453 years

Sol. $R = \dfrac{R_0}{2^{t/T_H}}$

$$2^{t/T_H} = \frac{R_0}{R} = \frac{16}{12} = \frac{4}{3}$$

$$\frac{t}{T_H} = \log\frac{4}{3}$$

$$t = \frac{T_H}{\log 2}[2\log 2 - \log 3]$$

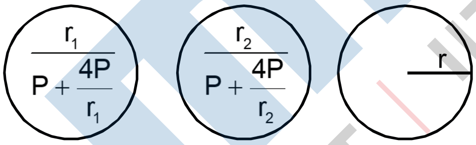
$$= \frac{5760\,y[2 \times 0.30 - 0.48]}{0.3010}$$

$$= \frac{5760\,y \times 0.12}{0.3} \approx 2391\,y$$

20. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S the change in total surface area, T is the surface tension and P atmospheric pressure, which of the following relation is correct?

(A) 4 PV + 3ST = 0 (B) 3PV + 2ST = 0 (C*) 3PV + 4ST = 0 (D) 2PV + 3ST = 0

Sol.



$$\left(P + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(P + \frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3 = \left(P + \frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$$PV + \frac{4T}{3}(4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2)$$

3pv + 4TS

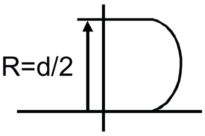
21. A positive charge 'q' of mass 'm' is moving along the + x axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then: :

(A) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{2v}$ (B) $B = \dfrac{mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

(C) $B = \dfrac{mv}{2qd}$ and $\Delta t = \dfrac{\pi d}{2v}$ (D*) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

Sol. $\dfrac{1.8\times10^{-3}\times10^{-4}}{2.5\times1.6\times10^{-19}}$

R=d/2



$\dfrac{mv^2}{r}=qvB$ $\qquad \dfrac{18}{2.5\times1.6}10^{12}$

$r=\dfrac{mv}{qB}$

$\dfrac{d}{2}=\dfrac{mv}{qB} \qquad \Rightarrow B=\dfrac{2mv}{qd}$

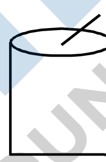
$\dfrac{\pi d}{2v}$

22. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is:

(A*) $\dfrac{\sqrt{2}A}{\pi a^2}\sqrt{\dfrac{h}{g}}$ (B) $\dfrac{A}{\sqrt{2}\pi a^2}\sqrt{\dfrac{h}{g}}$ (C) $\dfrac{2A}{\pi a^2}\sqrt{\dfrac{h}{g}}$ (D) $\dfrac{2\sqrt{2}A}{\pi a^2}\sqrt{\dfrac{h}{g}}$

Sol. $-Adh=a\cdot\sqrt{2gh}.dt$

$\int_H^0\dfrac{dh}{\sqrt{2gh}}=-\dfrac{q}{A}\int_0^t dt$



$\Rightarrow \dfrac{2}{\sqrt{2g}}\sqrt{h^{-1/2+1}} \quad \dfrac{2}{\sqrt{2g}}\left|\sqrt{h}\right|=\dfrac{q}{A}t$

$t=\dfrac{\sqrt{2}A}{\pi a^2}\sqrt{\dfrac{h}{g}}$

23. In an experiment of single slit diffraction pattern, first minimum for red light coincide with first maximum of some other wavelength. If wavelength of red light is 6600 Å, then wavelength of first maximum will be:

(A*) 3300 Å (B) 6600 Å (C) 5500 Å (D) 4400 Å

Sol. $\dfrac{\lambda_R b}{a}=\dfrac{3}{2}\lambda\dfrac{b}{a}$

$\lambda=\dfrac{2\lambda_R}{3}=\dfrac{2}{3}\times6600$

λ = 4400 Å

24. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly:

(A) 4.02 V/m (B) 1.34 V/m (C) 5.36 V/m (D*) 2.68 V/m

Sol. $I=\langle u\rangle c=\dfrac{P}{4\pi r^2}$

$\dfrac{\epsilon_0\epsilon_0^2}{2}C=\dfrac{P}{4\pi r^2}$

$$\epsilon_0^2 = \frac{P}{2\pi r^2 \epsilon_0 C}$$

$$\epsilon_0 = \sqrt{\frac{P}{2\pi r^2 \epsilon_0 C}} = \sqrt{\frac{3}{2\pi(5)^2 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$$

ϵ_0 = 2.68 volt/metre

25. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator?

(A) 45 s (B) 27 s (C) 37 s (D*) 24 s

Sol. $60 = \frac{d}{v_{man}}$

$40 = \frac{d}{v_{es}}$

$t = \frac{d}{v_{man} + v_{es}}$

$\frac{1}{t} = \frac{v_{man}}{d} + \frac{v_{es}}{d} = \frac{1}{60} + \frac{1}{40}$

t = 24 sec

26. A beam of light has two wavelengths 4972 Å and 6216 Å with a total intensity of 3.6 × 10^{-3} Wm^{-2} equally distributed among the two wavelengths. The beam falls normally on an area of 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photo electrons liberated in 2s is approximately:

(A) 6 × 10^{11} (B*) 9 × 10^{11} (C) 15 × 10^{11} (D) 11 × 10^{11}

Sol. $\frac{I\Delta t\lambda}{hc}$

$\frac{1.8 \times 10^{-3} \times 1 \times 10^{-4}}{2.5 \times 10^{-10}} \cdot 10^3$

$\frac{18}{25} \times 10^3$ photon

$\frac{1.8 \times 10^{-3} \times 1 \times 10^{-4}}{2 \times 10^{-10}}$

$\frac{12400}{6216} \cdot \frac{1.8}{2} \times 10^3$

27. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potentiometer wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of voltmeter will be:

(A) – 0.15 volt (B) 0.5 volt (C) 0.15 volt (D*) – 0.05 volt

Sol. Potential gradient

$$\eta = \frac{1.1}{440}$$

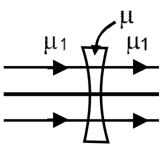
$$V_{actual} = \frac{1.1}{440} \times 220$$

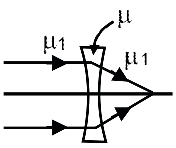
$$= 0.55 \text{ volt}$$

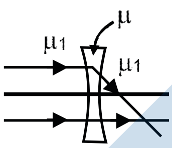
Error = 0.5 – 0.55

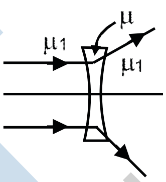
= – 0.05 volt

28. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is:

(A) 

(B*) 

(C) 

(D) 

Sol. bc = $\mu_1 > \mu$
So, option (3) is correct.

29. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of :

(A) 0.5 eV to 0.8 eV (B) 0.9 eV to 1.6 eV (C) 0.1 eV to 0.4 eV (D) 1.7 eV to 3.0 eV

Sol. $\frac{12400}{3000}$ to $\frac{12400}{7800}$ or 3.26eV to 1.6eV

30. For sky wave propagation, the radio waves must have a frequency range in between:

(A) 45 MHz to 50 MHz (B*) 5 MHz to 25 MHz (C) 35 MHz to 40 MHz (D) 1 MHz to 2 MHz

PART-B-CHEMISTRY

31. Aminoglycosides are usually used as:

(A) analgesic (B*) antibiotic (C) hypnotic (D) antifertility

Sol. Aminoglycosides is bactericidal antibiotic.

32. Which of the following xenon-OXO compounds may not be obtained by hydrolysis of xenon fluorides ?

(A) XeO_3 (B*) XeO_4 (C) $XeOF_4$ (D) XeO_2F_2

Sol. $XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

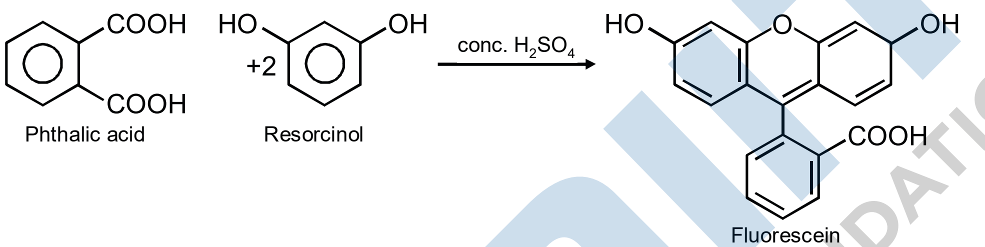
$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$

$XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

33. Phthalic acid reacts with resorcinol in the presence of concentrated H_2SO_4 to give:

(A) Phenolphthalein (B) Coumarin (C) Alizarin (D*) Fluorescein

Sol.



34. The conjugate base of hydrazoic acid is:

(A*) N_3^- (B) HN_3^- (C) N^{-3} (D) N_2^-

Sol. $N_3H \longrightarrow N_3^{1} + H^{\oplus}$

Hydrazoic conjugate lase Azide ion

35. Which of the following will not show mutarotation?

(A*) Sucrose (B) Glucose (C) Lactose (D) Maltose

Sol. Sucrose does not exhibit mutarotation because the glycosidic bond is between the anomeric carbon of glucose and anomeric carbon of fructose. Hence hemiacetal is not present in sucrose, while it is there in lactose, maltose and glucose.

36. How many electrons would be required to deposited 6.35 g of copper at the cathode during the electrolysis of an aqueous solution of copper sulphate? (Atomic mass of copper = 63.5 u, N_A = Avogadro's constant):

(A*) $\frac{N_A}{5}$ (B) $\frac{N_A}{2}$ (C) $\frac{N_A}{20}$ (D) $\frac{N_A}{10}$

Sol. $W = \frac{E}{96500} \times Q$

$\Rightarrow 6.35 = \frac{63.5}{2 \times 96500} \times Q$

$\Rightarrow$ Q = 2 × 9650 coulomb

$\Rightarrow$ 1F = charge of 1 mol of e^- = 96500

$$\therefore \text{ No. of } e^- = \frac{N_A}{10}\times 2$$

$$= \frac{N_A}{5}$$

37. The rate coefficient (k) for a particular reactions is 1.3 × 10^{-4} M^{-1} s^{-1} at 100ºC, and 1.3 × 10^{-3} M^{-1} s^{-1} at 150ºC. What is the energy of activation (E_A) (in kJ) for this reaction? (R = molar gas constant = 8.314 JK^{1} mol^{-1})

(A) 16 (B) 132 (C) 99 (D*) 60

Sol.
$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$\Rightarrow \quad \log\frac{1.3\times 10^{-3}}{1.3\times 10^{-4}} = \frac{E_a}{2.303R}\left[\frac{1}{373}-\frac{1}{423}\right]$$

$$\Rightarrow \quad 1 = \frac{E_a}{2.303R}\left[\frac{50}{373\times 423}\right]$$

$$\Rightarrow \quad E_a = \frac{2.303\times 8.314\times 373\times 423}{1000\times 50}$$

$$= 60.42 \text{ kJ}$$

38. Which one of the following complexes will most likely absorb visible light?
(At nos. Sc = 21, Ti = 22, V = 23, Zn = 30)

(A) $Zn(NH_3)_6]^{2+}$ (B) Ti $(NH_3)_6]^{4+}$ (C) $[Sc(H_2O)_6]^{3+}$ (D*) $[V(NH_3)_6]^{3+}$

39. Which one of the following exhibits the largest number of oxidation states?

(A) V(23) (B*) Mn(25) (C) Ti(22) (D) Cr(24)

Sol. Mn $\longrightarrow$ + 7 oxidation state

40. In the Victor-Meyer's test, the colour given by 1º 2º and 3º alcohols respectively.

(A) Red, blue, violet (B) Colourless, red, blue

(C*) Red, blue, colourless (D) Red, colourless, blue

Sol. Victor Meyer's Test :

1°Alcohol

$$RCH_2OH \xrightarrow[\text{or Red P + } I_2]{\text{Conc. HI}} RCH_2I \xrightarrow{AgNO_2} RCH_2NO_2 \xrightarrow[\text{or } HNO_2]{(NaNO_2 + H_2SO_2)} R\!-\!\underset{\underset{NaOH}{\|}}{C}\!-\!NO_2 \xrightarrow{NaOH} \text{Blood Red colour}$$

Nitrolic acid

2°Alcohol

$$R_2CHOH \xrightarrow[\text{or Red P + } I_2]{\text{Conc. HI}} R_2CHI \xrightarrow{AgNO_2} R_2CHNO_2 \xrightarrow[\text{or } HNO_2]{(NaNO_2 + H_2SO_2)} R_2\underset{\underset{NO}{|}}{C}\!-\!NO_2 \xrightarrow{NaOH} \text{Blue colour}$$

Pseudonitrol

3°Alcohol

$$R_3C\!-\!OH \xrightarrow[\text{or Red P + } I_2]{\text{Conc. HI}} R_3C\!-\!I \xrightarrow{AgNO_2} R_3C\!-\!NO_2 \xrightarrow[\text{or } HNO_2]{(NaNO_2 + H_2SO_2)} \text{No reaction} \xrightarrow{NaOH} \text{colourless}$$

41. Which of the following arrangements represents the increasing order (smallest to largest) of ionic radii of the given species O^{2-}, S^{2-}, N^{3-}, P^{3-}?

(A) $N^{3-} < O^{2-} < P^{3-} < S^{2-}$ (B) $O^{2-} < P^{3-} < N^{3-} < S^{2-}$

(C) $N^{3-} < S^{2-} < O^{2-} < P^{3-}$ (D*) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

Sol. $O^{2-} < N^{3-}$

$10e^{\ominus}$ $10e^{\ominus}$ isoelectronic

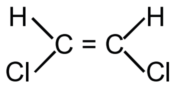
$S^{-2} < P^{3-}$

$O^{2-} < N^{3-} < S^{-2} < P^{3-}$

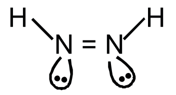
42. Which of the following molecules has two sigma(σ) and two pi(π) bonds ?

(A) $C_2H_2Cl_2$ (B) C_2H_4 (C) N_2F_2 (D*) HCN

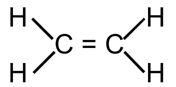
Sol. H–C≡N 2σ and 2π bond



5σ and 1π bond



3σ and 1π bond



5σ and 1π bond

43. Global warming is due to increase of:

(A) methane and O_3 in atmosphere (B) methane and nitrous oixde in atmosphere

(C) methane and CO in atmosphere (D*) methane and CO_2 in atmosphere

44. The de-Broglie wavelength of a particle of mass 6.63 g moving with a velocity of 100 ms^{-1} is:

(A) 10^{-35} m (B) 10^{-25} m (C*) 10^{-33} m (D) 10^{-31} m

Sol. $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34} \times 1000}{6.63 \times 100} = 10^{-33}$ m

45. Among the following species the one which causes the highest CFSE, Δo as a ligand is:

(A) CN^- (B) NH_3 (C) F^- (D*) CO

Sol. CO is the strongest ligand therefore it has maximum CFSE value.

46. Similarity in chemical properties of the atoms of elements in a group of the Periodic table is most closely related to :

(A*) number of valence electrons (B) number of principal energy levels

(C) atomic masses (D) atomic numbers

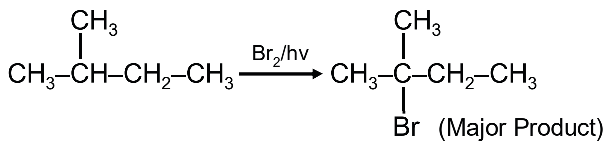
Sol. According to modern periodic law.

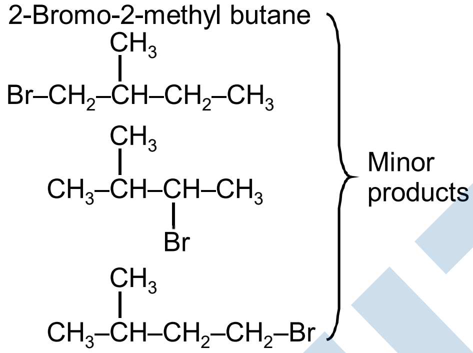
47. The major product obtained in the photo catalysed bromination of 2-methylbutane is:

(A) 1-bromo-3-methylbutane (B) 1-bromo-2-methylbutane

(C*) 2-bromo-2-methylbutane (D) 2-bromo-3-methylbutane

Sol.





relectivity ratio for bromination is

1° : 2° : 3° : : 1 : 82 : 1600

Hence 3° product will be major product.

48. Copper becomes green when exposed to moist air for a long period. This is due to:

(A) the formation of basic copper sulphate layer on the surface of the metal.

(B) the formation of a layer of cupric oxide on the surface of copper.

(C*) the formation of a layer of basic carbonate of copper on the surface of copper.

(D) the formation of a layer of cupric hydroxide on the surface of copper.

Sol. $CuCO_3$ $Cu(OH)_2$

Malachite green

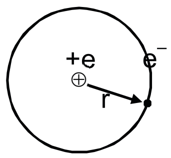
1 : 1 ratio.

49. If m and e are the mass and charge of the revolving electron in the orbit of radius r for hydrogen atom, the total energy of the revolving electron will be:

(A) $\frac{me^2}{r}$ (B) $\frac{e^2}{r}$ (C*) $-\frac{1}{2}\frac{e^2}{r}$ (D) $\frac{1}{2}\frac{e^2}{r}$

Sol. TE = PE + KE $\{\because \frac{mv^2}{r} = \frac{ke^2}{r^2}$

$= -\frac{ke^2}{r} + \frac{1}{2}mv^2$ $\therefore \frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r}\}$



$= -\frac{ke^2}{r} + \frac{1}{2}\frac{ke^2}{r}$

$= -\frac{1}{2}\frac{ke^2}{r}$

$= -\frac{1}{2}\frac{e^2}{r}$ [In CGS system K = 1]

50. Excited hydrogen atom emits light in the ultraviolet region at 2.47×10^{15} Hz. With this frequency, the energy of a single photon is :

($h = 6.63 \times 10^{-34}$ Js)

(A) 2.680×10^{-19} J (B) 6.111×10^{-17} J (C*) 1.640×10^{-18} J (D) 8.041×10^{-40} J

Sol. E = hv = Energy of single photon

$= 6.63 \times 10^{-34}$ J. sec × 2.47×10^{15} 1/sec.

$= 16.37 \times 10^{-19}$ J

$= 1.637 \times 10^{-18}$ J

51. Hydrogen peroxide acts both as an oxidising and as a reducing agent depending upon the nature of the reacting species. In which of the following cases H_2O_2 acts as a reducing agent in acid medium?

(A) SO_3^{2-} (B*) MnO_4^- (C) KI (D) $Cr_2O_7^{2-}$

Sol. $H_2O_2 + MnO_4^- \xrightarrow{\overset{\oplus}{H}} O_2 + Mn^{+2}$

52. The entropy (S°) of the following substances are:

CH_4(g) 186.2 JK^{-1} mol^{-1}

O_2(g) 205.0 J K^{-1} mol^{-1}

CO_2(g) 213.6 J K^{-1} mol^{-1}

$H_2O(\ell)$ 69.9 J K^{-1} mol^{-1}

The entropy change (ΔS°) for the reaction

$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(\ell)$ is

(A) – 37.6 J K^{-1} mol^{-1} (B) –108.1 J K^{-1} mol^{-1} (C) – 312.5 K^{-1} mol^{-1} (D*) – 242.8 J K^{-1} mol^{-1}

Sol. $CH_{4(g)} + 2\,O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(\ell)}$

186.2 205 213.6 69.9

$\Delta S° = \Sigma(s°)_p - \Sigma(s°)_R$

$= [213.6 + (2 \times 69.9)] - [186.2 + 2 \times 205]$

$= 353.4 - 596.2$

$= -242.8$ J/mol K

53. The amount of $BaSO_4$ formed upon mixing 100 mL of 20.8% $BaCl_2$ solution with 50 mL of 9.8% H_2SO_4 solution will be:

(Ba = 137, Cl = 35.5 S = 32, H = 1 and O = 16)

(A) 33.2 g (B) 30.6 g (C*) 11.65 g (D) 23.3 g

Sol. $BaCl_2 + H_2SO_4 \longrightarrow BaSO_4 + 2\,HCl$

100ml 50 ml $\frac{1}{20}$ml

20.8% 9.8%

20.8 gm 4.9 gm $\quad \frac{1}{20} \times 233$ gm

$\frac{1}{10}$ mol $\frac{4.9}{98} = \frac{1}{20}$ mol $= 11.65$ gm

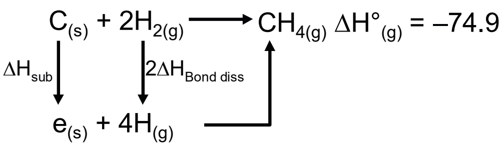
54. The standard enthalpy of formation ($\Delta_f H^\circ_{298}$) for methane, CH_4 is – 74.9 kJ mol^{-1}. In order to calculate the average energy given out in the formation of a C–H bond from this it is necessary to know which one of the following?

(A) the dissociation energy of the hydrogen molecule, H_2.

(B) the first four ionisation energies of carbon and electron affinity of hydrogen.

(C) the first four ionisation eergyies of carobn.

(D*) the dissociation energy of $H_2$and enthalpy of sublimation of carbon (graphite).

Sol. 

To calculate average bond energy of (C–H) bond, dissociation energy of H_2 and enthalpy of sublimation of carbon (graphite) is needed.

55. In the presence of peroxide, HCl and HI do not give anti-Markownikoff' addition to alkenes because:

(A) Both HCl and HI are strong acids

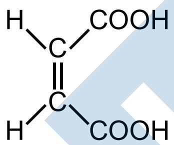
(B*) One of the steps is endothermic in HCl and HI

(C) HCl is oxidizing and the HI is reducing

(D) All the steps are exothermic in HCl and HI

56. Which one of the following acids does not exhibit optical isomerism?

(A) Tartaric acid (B) α-amino acids (C) Lactic acid (D*) Maleic acid

Sol. 

Maleic Acid

Maleic acid shows geometrical isomerism not optical isomerism.

57. In a monoclinic unit cell, the relation of sides and angles are respectively:

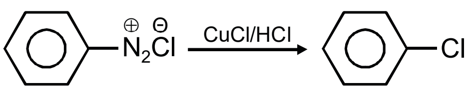
(A) $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$ (B*) $a \neq b \neq c$ and $\beta = \gamma = 90^\circ \neq \alpha$

(C) $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$ (D) $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$

58. Conversion of benzene diazonium chloride to chloro benzene is an example of which of the following reactions?

(A) Claisen (B) Wurtz (C*) Sandmeyer (D) Friedel-craft

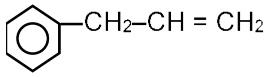
Sol. Sandmeyer reaction



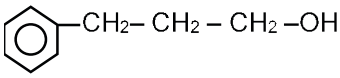
59. What happens when an inert gas is added to an equilibrium keeping volume unchanged?

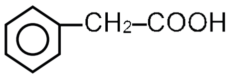
(A*) Equilibrium will remain unchanged (B) More product will form

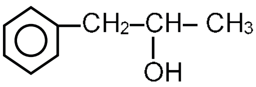
(C) More reactant will form (D) Less product will form

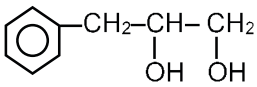
60. 

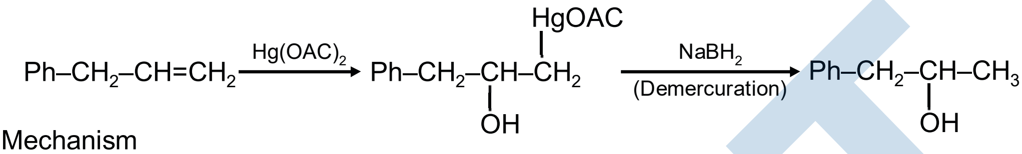
on mercuration demercuration produces the major product :

(A) 

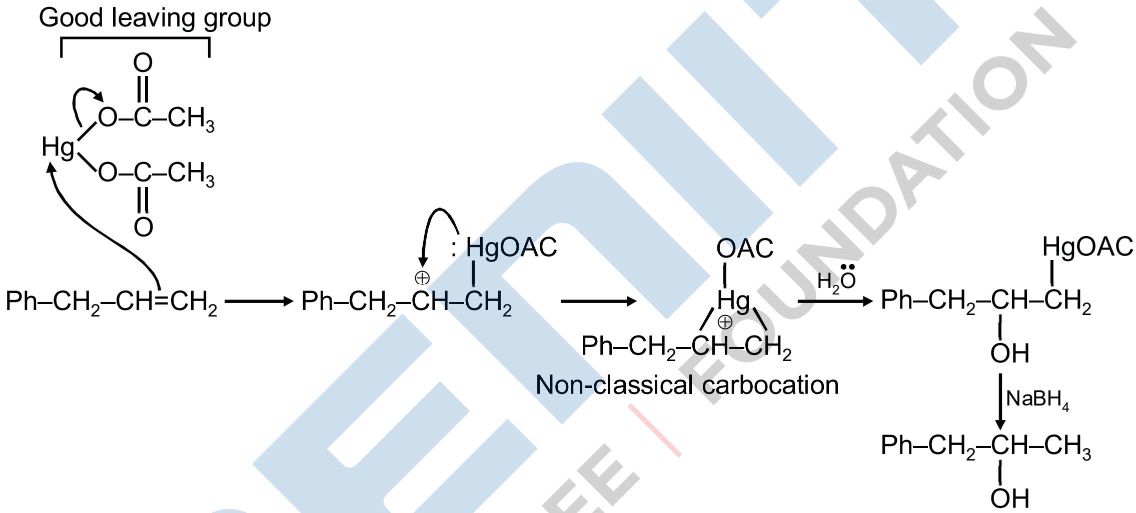
(B) 

(C*) 

(D) 

Sol. 

Mechanism



Rearrangement of carbocation formed is not possible due to formatiion of cyclic non-classical carbocation.

PART-C-MATHEMATICS

61. If $\hat{x}, \hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2$ is

(A) $\frac{3}{2}$ (B*) 3 (C) $3\sqrt{3}$ (D) 6

Sol. $|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2 = K$ let

$K = 2\hat{x}^2+2\hat{y}^2+2\hat{z}^2+2\hat{x}.\hat{y}+2\hat{y}.\hat{z}+2\hat{z}.\hat{x}$

$K = 2+2+2+2[\cos\theta_1+\cos\theta_2+\cos\theta_3]$

When $\theta_1=\theta_2=\theta_3=\frac{2\pi}{3}$

Then $K_{min} = 6+2\left(-\frac{3}{2}\right) = 6-3 = 3$

62. Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. $\frac{1}{M}$ If : G is 4: 5, then a: b can be

(A) 2: 3 (B*) 1 :4 (C) 1: 2 (D) 3: 4

63. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and A and B are respectively the maximum and the minimum values of f(θ), then (A, B) is equal to

(A) $\left(2+\sqrt{2}, -1\right)$ (B*) $\left(2+\sqrt{2}, 2-\sqrt{2}\right)$ (C) (3, – 1) (D) $\left(4, 2-\sqrt{2}\right)$

Sol. Expanding the determinant, we get

f(θ)= 1(1 + sinθcosθ) + cosθ(sinθ + cosθ) + 1(1-sin2θ)

= 1 + 2sinθcosθ + 2cos2θ

= 1 + sin2θ + (1 + cos2θ)

= 2 + sin2θ + cos2θ

Now,

sin2θ + cos2θ lies between $-\sqrt{2}$ to $\sqrt{2}$

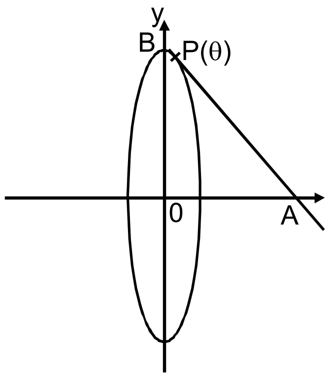
$\left[\sqrt{2}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right) \to \pm\sqrt{2}\right]$

$\therefore A = 2+\sqrt{2}:\quad B = 2-\sqrt{2}$

64. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81} = 1$ and the coordinate axes is

(A) 26 (B) 12 (C) 18 (D*) 36

Sol. Let P(4cosθ, 9sinθ) be a point on ellipse equation of tangent



$$\frac{x}{4}\cos\theta + \frac{y}{9}\sin\theta = 1$$

Let A & B are point of intersection of tangent at P with co-ordinate axes.

$$A\left(\frac{4}{\cos\theta}, 0\right) B\left(0, \frac{9}{\sin\theta}\right)$$

Area of ΔOAB = $\frac{1}{2}\left(\frac{4}{\cos\theta}\right)\left(\frac{9}{\sin\theta}\right) = \frac{36}{\sin 2\theta}$

65. If $\begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then

(A*) y = 2x (B) y = – x (C) y = x (D) y = – 2x

Sol. $AB = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

y + 3x = 6

3y – x = 6

y + 3x = 3y – x

4x = 4y

$\boxed{x = y}$

66. If a line intercepted between the coordinate axes is trisected at a point A (4, 3), which is nearer to x-axis, then its equation is

(A) 3x + 8y = 36 (B) x + 3y = 13 (C) 4x – 3y = 7 (D*) 3x + 2y = 18

Sol. $4 = \frac{a}{3} \quad a = 12$

$3 = \frac{2b}{3} \quad b = \frac{9}{2}$

$\frac{x}{12} + \frac{y}{9}(2) = 1$

$\boxed{3x + 8y = 36}$

67. The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^3 x + \cos^3 x\right)^2} dx$ is equal to

(A) $\frac{1}{(1+\cot^3 x)} + C$ (B*) $\frac{-1}{3\,(1+\tan^3 x)} + C$ (C) $\frac{\sin^3 x}{(1+\cos^3 x)} + C$ (D) $\frac{-\cos^3 x}{3\,(1+\sin^3 x)} + C$

Sol. $\int \frac{\sin^2 x \cos^2 x\, dx}{\left(\sin^3 x + \cos^3 x\right)^2} = \int \frac{\tan^2 x \sec^2 x\, dx}{\left(1+\tan^3 x\right)^2}$

Put $\tan^3 x = t$

$3\tan^2 x.\sec^2 x dx = dt$

$$\int \frac{dt}{3(1+t)^2} = \frac{-1}{3(1+t)} + C$$

$$= \frac{-1}{3(1+\tan^3 x)} + C$$

68. If for a continuous function f(x), $\int_{-\pi}^{t} (f(x)+x)\,dx = \pi^2 - t^2$, for all $t \geq -\pi$, then $f\left(\frac{-\pi}{3}\right)$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D*) π

Sol. $\int_{-\pi}^{t} (f(x)+x)dx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx + \int_{-\pi}^{t} x\,dx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \frac{3}{2}(\pi^2 - t^2)$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \int_{-\pi}^{t} -3x dx \Rightarrow f(x) = -3x$

$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi$

69. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i (1+x)^i$, for all x in R, then a_2 is

(A*) – 4 (B) – 8 (C) 6 (D) 10

Sol. $1 + x^4 + x^5 = a_0 + a_1(1 + x) + a_2(1 + x)^2 + a_3(1 + x)^3 + a_4(1 + x)^4 + a_5(1 + x)^5$

$= a_0 + a_1(1 + x) + a_2(1 + 2x + x^2) + a_3(1 + 3x + 3x^2 + x^3) + a_4(1 + 4x + 6x^2 + 4x^3 + x^4) + a_5(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

So, Coeff. of xi in LHS = Coeff. of xi on RHS

i = 5 $\Rightarrow 1 = a_5$ …(i)

i = 4 $\Rightarrow 1 = a_4 + 5a_5 = a_4 + 5$

$\Rightarrow a_4 = -4$ …(ii)

i = 3 $\Rightarrow 0 = a_3 + 4a_4 + 10a_5$

$\Rightarrow a_3 - 16 + 10 = 0$

$\Rightarrow a_3 = 6$ …(iii)

i = 2 $\Rightarrow 0 = a_2 + 3a_3 + 6a_4 + 10a_5$

$\Rightarrow a_2 + 18 - 24 + 10 = 0$

$\Rightarrow a_2 = -4$

Put x = – 1

$1 = a_0$

Now differentiate w.r.t. x.

$4x^3 + 5x^4 = a_1 + 2a_2(1 + x) + 3a_3(1 + x)^2 + ...$

Put x = – 10

$\Rightarrow 1 = a_1$

Again differentiate w.r.t. x

$12x^2 + 20x^3 = 2xa_2 + 6a_3 (1 + x)$

Put x = –1

$12 - 20 = 2a_2 \Rightarrow a_2 = -4$

70. The general solution of the differential equation, $\sin 2x \left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is

(A) $y\sqrt{\tan x} = \cot x + C$ (B) $y\sqrt{\tan x} = x + C$

(C) $y\sqrt{\cot x} = \tan x + C$ (D*) $y\sqrt{\cot x} = x + C$

Sol. $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

$\frac{dy}{dx} - \frac{y}{\sin 2x} = \sqrt{\tan x}$

$I.F. = e^{-\int \operatorname{cosec} 2x dx} = e^{-\frac{1}{2}\ln \tan x} = \frac{1}{\sqrt{\tan x}}$

$\Rightarrow$ General solution

$y.\frac{1}{\sqrt{\tan x}} = \int \sqrt{\tan x}.\frac{1}{\sqrt{\tan x}} + c$

$y\sqrt{\cot x} = x + c$

71. A number x is chosen at random from the set {1, 2, 3, 4, ……, 100}. Define the event: A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \ge 0$. Then P(A) is

(A) 0.20 (B) 0.70 (C*) 0.71 (D) 0.51

Sol. S = {1, 2, 3...........100}

A : Chosen no. x satisfies

$\frac{(x-10)(x-50)}{(x-30)} \ge 0$

$\therefore x \in \{10, 11, 12......29\} \cup \{50, 51.....100\}$

$P(A) = \frac{71}{100} = 0.71$

72. Let $z \ne -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $\left(z + \frac{1}{z}\right)$ is

(A*) any non-zero real number (B) a purely imaginary number

(C) any non-zero real number other than 1 (D) 0

Sol. Let Z = x + iy

$\frac{z-i}{z+i}$ is a purely imaginary number

$\Rightarrow \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y-1)}{x-i(y+1)}$ is a purely imaginary

$\Rightarrow \frac{(x^2+y^2-1)-i(2x)}{x^2+(y+1)^2}$ is purely imaginary

$\Rightarrow x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$ ...(i)

$z+\frac{1}{z} = x+iy+\frac{1}{x+iy}$

$= (x+iy)+\frac{1}{(x+iy)} \times \frac{(x-iy)}{(x-iy)}$

$= (x+iy)+\frac{(x-iy)}{x^2+y^2} = 2x$

$x \neq 1$

($\because$ if x = 1 then y = 0, from (i) & z won't be complex number)

If $x = 1 \Rightarrow y = 0$

$\Rightarrow z = 1$

$\Rightarrow \frac{z-i}{z+i} = \frac{1-i}{1+i}$ cannot be purely imaginary.

73. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = K\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, $\lambda \neq 0$, then k is equal to

(A) $-4\lambda abc$ (B*) $4\lambda^2$ (C) $-4\lambda^2$ (D) $4\lambda abc$

Sol. $R_2 \to R_2 - R_1$, $R_1 \to R_1 - R_3$

$\begin{vmatrix} \lambda(2a-\lambda) & \lambda(2b-\lambda) & \lambda(2c-\lambda) \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$

$= R_3 \to R_3 + R_1$, $R_1 \to R_1 - \frac{1}{2}R_2$

$= \begin{vmatrix} -\lambda^2 & -\lambda^2 & -\lambda^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= -4\lambda^3 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= +4\lambda^3 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$\therefore K = 4\lambda^2$

74. Let $\bar{X}$ and M.D. be the mean and the mean deviation about $\bar{X}$ of n observations x_i, i = 1, 2, ……., n. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are

(A*) $\bar{X} + 5$, M.D. (B) $\bar{X} + 5$, M.D + 5 (C) $\bar{X}$, M.D. + 5 (D) $\bar{X}$, M.D.

Sol. If all the observations are increased by K then mean is increased by K but M.D. remains same.

75. The least positive integer n such that $1 - \frac{2}{3} - \frac{2}{3^2} - \ldots\ldots. - \frac{2}{3^{n-1}} < \frac{1}{100}$, is

(A) 4 (B*) 6 (C) 5 (D) 7

Sol. $1 - 2\left(\frac{1}{3^1} + \frac{1}{3^2} \ldots\ldots \frac{1}{3^{n-1}}\right) < \frac{1}{100}$

$$1 - 2\left(\frac{1}{3}\right)\frac{\left[1 - \frac{1}{3^{n-1}}\right]}{\left(\frac{2}{3}\right)} < \frac{1}{100}$$

$$\Rightarrow 1 - 1 + \frac{1}{3^{n-1}} < \frac{1}{100}$$

$100 < 3^{n-1}$

$n - 1 = 5$

$n = 6$

76. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is

(A) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$ (B) $(p \vee q) \Rightarrow r$ (C*) $(p \Rightarrow q) \vee (p \Rightarrow r)$ (D) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$

Sol. $p \rightarrow (q \vee r)$

$\sim p \vee (q \vee r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$(p \Rightarrow q) \vee (p \Rightarrow r)$

77. If $\left(2 + \frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are

(A) 28^{th} and 29^{th} (B) 27^{th} and 28^{th} (C) 7^{th} and 8^{th} (D*) 8^{th} and 9^{th}

Sol. $\left(2 + \frac{x}{3}\right)^{55}$

General term

$${}^{55}C_r \times 2^{55-r} \times \left(\frac{x}{3}\right)^r$$

Let T_{r+1} and T_{r+2} are having some co-efficients

$\Rightarrow$ Coff. of T_{r+1} = Coff. of T_{r+2}

$${}^{55}C_r \times 2^{55-r} \times \left(\frac{x}{3}\right)^r = {}^{55}C_{r+1} \times 2^{54-r} \times \left(\frac{1}{3}\right)^{r+1}$$

$\Rightarrow r = 6$

$\Rightarrow$ Coff. of T_7 = Coff. of T_8

78. For the two circle $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is/are

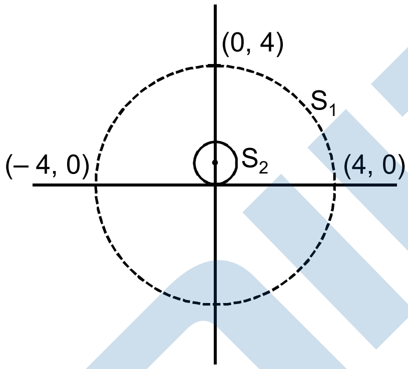
(A) two pairs of common tangents
(B) one pair of common tangents
(C*) no common tangent
(D) three common tangents

Sol. S-I : $x^2 + y^2 = 42$

S-II : $x^2 + (y - 1)^2 = 1$

Aliter : Distance between their centres is 1 units and sum of their radii is 3 so one of them lie completely inside the other, hence no common tangent.

Also, from figure we can say no common tangent :



79. Let f,g : $R \to R$ be two functions defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and g(x) = x f(x)

Statement-I: f is a continuous function at x = 0.

Statement-II: g is differentiable function at x = 0.

(A) Statement-I is false, Statement-II is true.
(B) Both Statements I and II are false
(C) Statement-I is true, statement-II is false.
(D*) Both Statement-I and II are true.

Sol. **Statement-I**

$$f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0 = f(0)$$

Hence f(x) is continuous function

$$g(0) = 0 = \lim_{x\to 0} g(x)$$

<u>**LHD :**</u>

$$\lim_{x\to 0} \frac{g(0-h) - g(0)}{-h}$$

$$\lim_{x\to 0}\frac{h^2\sin\left(-\frac{1}{h}\right)-0}{-h}$$

LHD = 0

RHD :

$$\lim_{h\to 0}\frac{g(0+h)-g(0)}{h}$$

$$\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}$$

RHD = 0

LHD = RHD

Hence g(x) at x = 0 is diff. function.

80. Two tangents are drawn from a point (– 2, – 1) to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan\alpha|$ is equal to

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C*) 3 (D) $\frac{1}{3}$

Sol. Let equation of tangent from (– 2, –1) be

y + 1 = m (x + 2)

$\Rightarrow$ y = mx + (2m – 1)

Condition of tangency, $C = \frac{a}{m}$

i.e., $2m - 1 = \frac{1}{m}$

$\Rightarrow 2m^2 - m - 1 = 0$

(2m + 1) (m – 1) = 0

$m = -\frac{1}{2}, 1$

Now, $|\tan\alpha| = \left|\frac{m_1+m_2}{1+m_1m_2}\right| = \left|\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right| = 3$

81. If the distance between planes, 4x – 2y – 4z + 1 = 0 and 4x – 2y – 4z + d = 0 is 7, then d is

(A*) – 41 or 43 (B) – 42 or 44 (C) 42 or – 43 (D) 41 or – 42

Sol. $\left|\frac{d-1}{\sqrt{4^2+2^2+4^2}}\right| = 7 \Rightarrow \left|\frac{d-1}{6}\right| = 7$

d– 1 = ± 42

d = + 43, – 41

82. The sum of the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is

(A) 2 (B) – 2 (C) $-\sqrt{2}$ (D*) $\sqrt{2}$

Sol. $x^2 + |2x - 3| - 4 = 0$

Case-1 : $x \geq \frac{3}{2}$

$\Rightarrow x^2 + 2x - 3 - 4 = 0$

$\Rightarrow x = -1 \pm 2\sqrt{2}$

$\Rightarrow x = -1 + 2\sqrt{2}$

Case-2 : $x < \frac{3}{2}$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = 1 \pm \sqrt{2}$

$\Rightarrow x = 1 - \sqrt{2}$

$\Rightarrow \text{sum} = \sqrt{2}$

83. A symmetrical form of the line of intersection of the planes x = ay + b and z = cy + d is

(A) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$ (B) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$

(C*) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$ (D) $\frac{x-a}{b} = \frac{y-0}{1} = \frac{z-c}{d}$

Sol. $\frac{x-b}{a} = y = \frac{z-d}{c}$

$\Rightarrow \frac{x-b}{a} - 1 = y - 1 = \frac{z-d}{c} - 1$

$\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

84. If [ ] denotes the greatest integer function, then the integral $\int_0^{\pi} [\cos x]\, dx$ is equal to

(A) 0 (B) $\frac{\pi}{2}$ (C*) $\frac{-\pi}{2}$ (D) – 1

Sol. $\int_0^{x} [\cos x] dx$

$= \int_0^{\pi/2} 0.dx + \int_{\frac{\pi}{2}}^{\pi} -1 dx$

$= [-x]_{\frac{\pi}{2}}^{\pi} = -\frac{\pi}{2}$

85. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do not occupy odd places, is

(A*) 120 (B) 48 (C) 160 (D) 60

Sol. No. of ways of selecting 3 odd places out of 4 odd places.

$^4C_3 \times \frac{3!}{2!} \times \frac{5!}{3!2!}$

= 4 × 3 × 5 × 2

= 120

86. A relation on the set A = {x: | x | < 3, x $\in$ Z}, where Z is the set of integers is defined by R = {(x, y): y = | x |, x $\neq$ – 1}. Then the number of elements in the power set of R is

(A) 16 (B) 64 (C) 32 (D) 8

Sol. A = {– 2, – 1, 0, 1, 2}

R = {(– 2, 2) (0, 0) (1, 1), (1, 2)}

$n(P(R)) = 2^4 = 16$

87. If $f(x) = x^2 - x + 5,\ x > \frac{1}{2}$ and g(x) is its inverse function, then g'(7) equals

(A) $\frac{-1}{13}$ (B) $\frac{1}{13}$ (C*) $\frac{1}{3}$ (D) $\frac{-1}{3}$

Sol. $f(x) = x^2 - x + 5$

g(f(x)) = x

$g'(f(x)) = \frac{1}{f'(x)}$

$\Rightarrow g'(f(2)) = \frac{1}{f'(2)}$

$\Rightarrow g'(7) = \frac{1}{3}$

88. Statement-I: The equation

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$.

Statement–II: For any x $\in$ R,

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ and $0 \leq \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$

(A*) Both Statement-I and II are true.

(B) Statement-I is true, statement-II is false.

(C) Both Statements I and II are false

(D) Statement-I is false, Statement-II is true.

Sol. **Statement-I**

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$

$\Rightarrow (\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3$

$\Rightarrow (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x) = a\pi^3$

$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi}{2}\left(\frac{\pi}{2} - \cos^{-1}x\right)\cos^{-1}x = a\pi^3$

$$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi^2}{2}\cos^{-1}x + \frac{3\pi}{2}(\cos^{-1}x)^2 = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[(\cos^{-1}x)^2 - \frac{\pi}{2}\cos^{-1}x\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[\left(\cos^{-1}x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\left(\cos^{-1}x - \frac{\pi}{4}\right)^2 = \left(\frac{2a}{3} - \frac{1}{48}\right)\pi^2 \qquad ...(1)$$

$\because 0 \le \cos–1x \le \pi$

$$-\frac{\pi}{4} \le \left(\cos^{-1}x - \frac{\pi}{4}\right) \le \frac{3\pi}{4}$$

$$0 \le \left(\cos^{-1}x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$$

From equation (1)

$$\frac{1}{32} \le a \le \frac{7}{8}$$

$\therefore$ Statement-I is false

Statement-II

sin–1x + cos–1x $= \frac{\pi}{2} \forall x \in [-1,1]$

not for any $x \in R$ so Statement-II is false

89. If the three distinct lines x + 2ay + a = 0, x + 3by + b = 0 and x + 4ay + a = 0 are concurrent, then the point (a, b) lies on a

(A) hyperbola (B) parabola (C*) straight line (D) circle

Sol. x + a(2y + 1) = 0

x + b(3y + 1) = 0

x + a(4y + 1) = 0

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 2a & a \end{vmatrix} = 0$$

$\Rightarrow$ 2a(b – a) = 0

2a = 0 or b = a

Locus of (a, b) $\Rightarrow$ x = 0 or y = x

90. Let f and g be two differentiable functions on R such that $f'(x) > 0$ and $g'(x) < 0$, for all $x \in R$. Then for all x

(A) $g(f(x)) < g(f(x + 1))$
(B*) $f(g(x)) > f(g(x - 1))$
(C) $g(f(x)) > g(f(x - 1))$
(D) $f(g(x)) > f(g(x + 1))$

Sol. $f'(x) > 0 \Rightarrow f(x)$ is increasing function

$g'(x) < 0 \Rightarrow g(x)$ is decreasing function

Now,

(i) $x > x - 1$

$f(x) > f(x - 1)$

$g(f(x)) < g(f(x - 1))$

and

(ii) $x + 1 > x$

$f(x + 1) > f(x)$

$g(f(x + 1)) < g(f(x))$

(iii) $x > x - 1$

$g(x) < g(x - 1)$

$f(g(x)) < f(g(x - 1))$

(iv) $x + 1 > x$

$g(x + 1) < g(x)$

$f(g(x + 1)) < f(g(x))$